Marginal Structural Mean Model

$$E(Y_a \mid M) = m(a, M; \beta)$$

For example

$$E(Y_a \mid M) = \beta_0 + \beta_1 a + \beta_2 M + \beta_3 aM$$

How do we estimate β_2?

Case 1: $M = L$, then

$$E(Y_a \mid M) = E(Y \mid A = a, M)$$

Then, our model is just a standard regression model

$$E(Y \mid A, M) = \beta_0 + \beta_1 A + \beta_2 M + \beta_3 AM$$

which we fit by least squares

Case 2: If $M \neq L$, then

$$E(Y_a \mid M) \neq E(Y \mid A=a, M)$$

So it is not true that $E(Y \mid A, M) = \beta_0 + \beta_1 A + \beta_2 M + \beta_3 AM$

However, if we weight each treated person by $1/P(A=1 \mid L)$

each untreated person by $1/P(A=0 \mid L)$

we can pretend as if everybody was assigned to $A=1$ and everybody was assigned to $A=0$

Now we have a pseudo-study in which the risk factors are balanced in both arms.

So we can pretend that in the pseudo study

$$E(Y \mid A = a, M) = E(Y_a \mid M)$$

and now we can just estimate β by least squares.

So the bottom line is that to estimate β when

M ≠ L we do it by weighted least squares with weights equal to $\frac{1}{P(A|L)}$