$$E(Y_1) = \sum_{\ell} E(Y \mid A=1, L=\ell)\, P(L=\ell) =$$

$$= E\left[E(Y \mid A=1, L) \right]$$

$$E(Y_1) = \int E(Y \mid A=1, L=\ell)\, f_L(\ell)\, d\ell$$

$$E(Y \mid A=1, L=\ell) = g(\ell) \quad ; \quad E(Y_1) = E\left[g(L)\right]$$

$$\frac{1}{m}\sum_{i=1}^{m} g(L_i) = \hat{E}(Y_1)$$

Formulate a model

$$E(Y \mid A=1, L=l) = \alpha_0 + \alpha_1 l + \alpha_2 l^2$$

Fit the model by least squares $\to$ get $\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2$

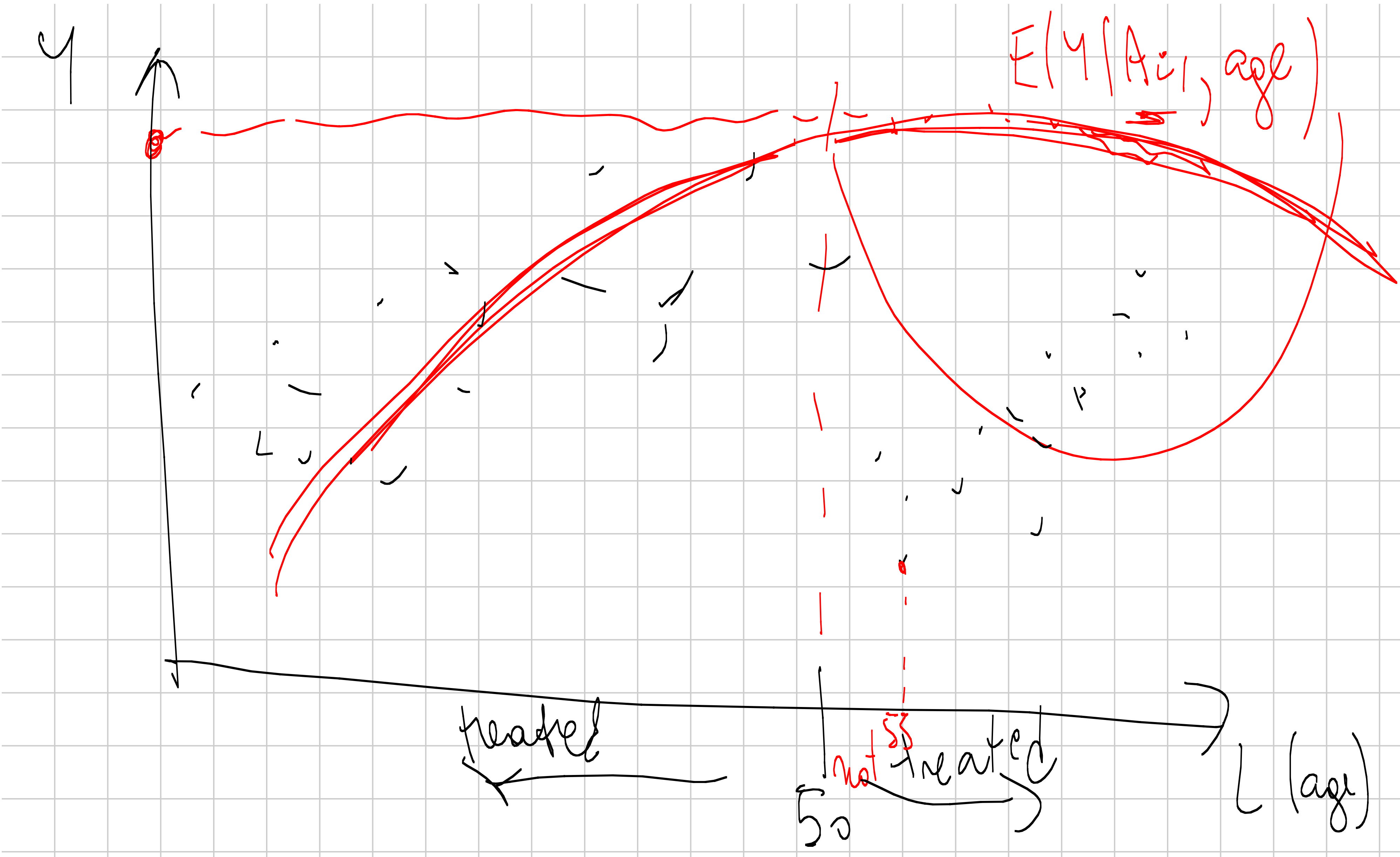
For every L_i I obtain

$$\hat{E}(Y \mid A=1, L_i) = \hat{\alpha}_0 + \hat{\alpha}_1 L_i + \hat{\alpha}_2 L_i^2 = \hat{g}(L_i)$$

Then we estimate $E(Y_1)$ with

$$\hat{E}(Y_1) = \frac{1}{m} \sum_{i=1}^{n} \hat{g}(L_i)$$

$$= \frac{1}{m} \sum_{i=1}^{n} \left[\hat{\alpha}_0 + \hat{\alpha}_1 L_i + \hat{\alpha}_2 L_i^2 \right]$$



Y
E(Y | Ai 1, age)
treated
50
not treated
53
L (age)

	A	L	Y	weight
John	1	60	.	$1/0.001 = 1000$
Pet	1	45	.	$1/0.25 = 4$
Mark	1	30	.	
Mary	0	20		$\frac{1}{0.0001} = 10{,}000$
Lanney	0			

weight = $1/P(A \mid L)$

$$\log\left(\frac{P(A=1 \mid L)}{P(A=0 \mid L)}\right) = \delta_0 + \delta_1 L + \delta_2 L^2$$

Fit model by logistic regression

$\rightarrow$ get $\hat{\delta}_0$, $\hat{\delta}_1$ and $\hat{\delta}_2$

$$\hat{P}(A=1 \mid L=60) = \frac{\exp\left(\hat{\delta}_0 + \hat{\delta}_1 60 + \hat{\delta}_2 60^2\right)}{1 + \exp\left(\hat{\delta}_0 + \hat{\delta}_1 60 + \hat{\delta}_2 60^2\right)} = 0.0001$$

$$\hat{P}(A=1 \mid L=45) = \frac{\qquad\qquad 45 \qquad\qquad 45^2}{\underline{\quad} \qquad \underline{\qquad} \qquad \underline{\quad}} = 0.25$$

$$\hat{P}(A=0 \mid L=20) = \frac{1}{1 + \exp\left(\hat{\delta}_0 + \hat{\delta}_1 20 + \hat{\delta}_2 20^2\right)} = 0.0001$$

$$E(Y_a \mid age) = \beta_0 + \beta_1 a + \beta_2\, age + \beta_3\, age \cdot a$$

$$E(Y_1 \mid age) = \beta_0 + \beta_1 + (\beta_2 + \beta_3)\, age$$

$$E(Y_0 \mid age) = \beta_0 + \beta_2\, age$$

$$ATE(age) = \boxed{\beta_1 + \beta_3\, age}$$

$$E(Y_0 \mid age) = \beta_0 + \beta_2\, age$$

Regress Y on age among the untreated

$$E(Y \mid A=0, age)$$

Fit model

$$E(Y \mid A=a, \text{age}) = \beta_0 + \beta_1 A + \beta_2 \,\text{age} + \beta_3 A \cdot \text{age}$$

$$\not\perp\!\!\!\perp$$

$$E(Y_a \mid \text{age})$$